A posteriori Parameter Selection for Local Regularization

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Abstract

The development of local regularization methods stemmed from the theoretical justification of a generalization by P. K. Lamm in 1995 of the practical method due to J. V. Beck for solving the discretized inverse heat conduction problem. Since then the convergence theory associated with a priori parameter selection has evolved to include finitely smoothing linear Volterra problems, nonlinear Hammerstein and autoconvolution problems, as well as linear non-Volterra integral equations such as those arising in image processing.

In recent years, we advanced the development of this theory through the construction of an a posteriori parameter selection principle for local regularization which is theoretically justified and suitable for L^p data as well as for data smoothed in some way. We will present these results, give rates of convergence under suitable source conditions, and illustrate effectiveness of the principle with some numerical examples.

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